

Summary of π – π Scattering Experiments

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Abstract: The $\pi\pi$ scattering amplitude at threshold is fully determined by the chiral symmetry breaking part of the strong interaction, and, thus, directly constrains the form of the low energy effective lagrangians. Current status of the study of the low energy $\pi\pi$ interaction is discussed, particularly the recent results on reactions $\pi N \rightarrow \pi\pi N$ near threshold. Present levels of experimental uncertainties and limitations inherent to the available analysis methods leave ample room for improvements in the determination of the s-wave $\pi\pi$ scattering lengths. Experimental improvements are expected from new measurements of K_{e4} decays and from attempts to study $\pi^+\pi^-$ atoms, while further theoretical work is required in order to make full use of the extensive near-threshold $\pi N \rightarrow \pi\pi N$ data that has recently become available.

1 Introduction

Effects of chiral symmetry breaking in low energy interactions of pions, and light hadrons in general, have been studied for over thirty years. Strong interactions break chiral symmetry both “spontaneously” and explicitly. Spontaneous breaking of chiral symmetry is well understood and has led to new concepts and methods that have transcended the domain of intermediate energy physics. On the other hand, the precise mechanism of explicit chiral symmetry breaking (ChSB) remained largely unresolved until the establishment of QCD as the correct theory of strong interactions. The study of explicit ChSB, however, remains relevant even today, owing to the failure of the full QCD to describe strong interaction phenomena at energies below a few GeV. At these energies, QCD becomes nonperturbative and intractable in practice by available calculational methods. In order to overcome this problem, a broad theoretical effort is under way to develop phenomenological lagrangian models based solely on the symmetry properties of the full

QCD, as suggested by Weinberg [1]. Chiral symmetry plays a particularly important role at low energies, since it is violated only slightly in the $SU(2)$ realization, and is essential for the understanding of the lightest hadrons. For this reason, chiral symmetry provides either the basic framework or the essential constraints for all modern effective lagrangian models at low energies, such as the chiral perturbation theory (ChPT) [2], and the various realizations of the Nambu–Jona-Lasinio model [3].

It has been long established [4] that low energy $\pi\pi$ scattering provides a particularly sensitive tool in studying the explicit breaking of chiral symmetry, since $a(\pi\pi)$, the $\pi\pi$ scattering lengths, vanish exactly in the chiral limit. To the extent that they differ from zero, $a(\pi\pi)$ provide a direct measure of the symmetry breaking term in the pion sector. Stated in more contemporary language, detailed knowledge of the $\pi\pi$ interaction scattering lengths and phase shifts provides much needed input in fixing the parameters of ChPT and other effective models.

2 Experimental Determination of $\pi\pi$ Scattering Lengths

Experimental evaluation of $\pi\pi$ scattering observables is difficult, primarily because free pion targets are not available. Scattering lengths are especially hard to determine since they require measurements close to the $\pi\pi$ threshold, where the available phase space strongly limits measurement rates. Over the years several reactions have been studied or proposed as a means to obtain near-threshold $\pi\pi$ phase shifts, such as $\pi N \rightarrow \pi\pi N$, K_{e4} decays, $\pi^+\pi^-$ atoms, $e^+e^- \rightarrow \pi\pi$, etc. In practice, only the first two reactions have so far proven useful in studying threshold $\pi\pi$ scattering. The three main methods are discussed below in the order of decreasing reliability.

2.1 Analysis of $K \rightarrow \pi^+\pi^-e^+\nu$ decays

The $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay (called K_{e4}) is in several respects the most suitable process for the study of near-threshold $\pi\pi$ interactions. The interaction takes place between two real pions on the mass shell, the only hadrons in the final state. The dipion invariant mass distribution in K_{e4} decay peaks close to the $\pi\pi$ threshold, and $l = I = 0$ and $l = I = 1$ are the only dipion quantum states contributing to the process. These factors, and the well understood $V - A$ nature of the weak decay, favor the K_{e4} process among all others in terms of theoretical uncertainties. On the other hand, measurements are impeded by the low branching ratio of the decay, $\sim 3.9 \times 10^{-5}$.

The most recent K_{e4} measurement was made by a Geneva–Saclay collaboration in the mid-1970’s [5]. Using a detector system consisting of pion

Čerenkov counters, wire chambers, a bending magnet and plastic scintillator hodoscopes around a 4 m decay zone, Rosselet and coworkers detected some 30,000 K_{e4} decays. Analysis of this low-background, high-statistics data illustrates well the difficulties encountered in extracting experimental $\pi\pi$ scattering lengths. Figure 1. summarizes the $\pi\pi$ phase shift information below 400 MeV derived from all K_{e4} measurements to date. The curves in Fig. 1 correspond to three different values of $a_0^0(\pi\pi)$, and illustrate the relative insensitivity of the data at the present level of experimental accuracy.

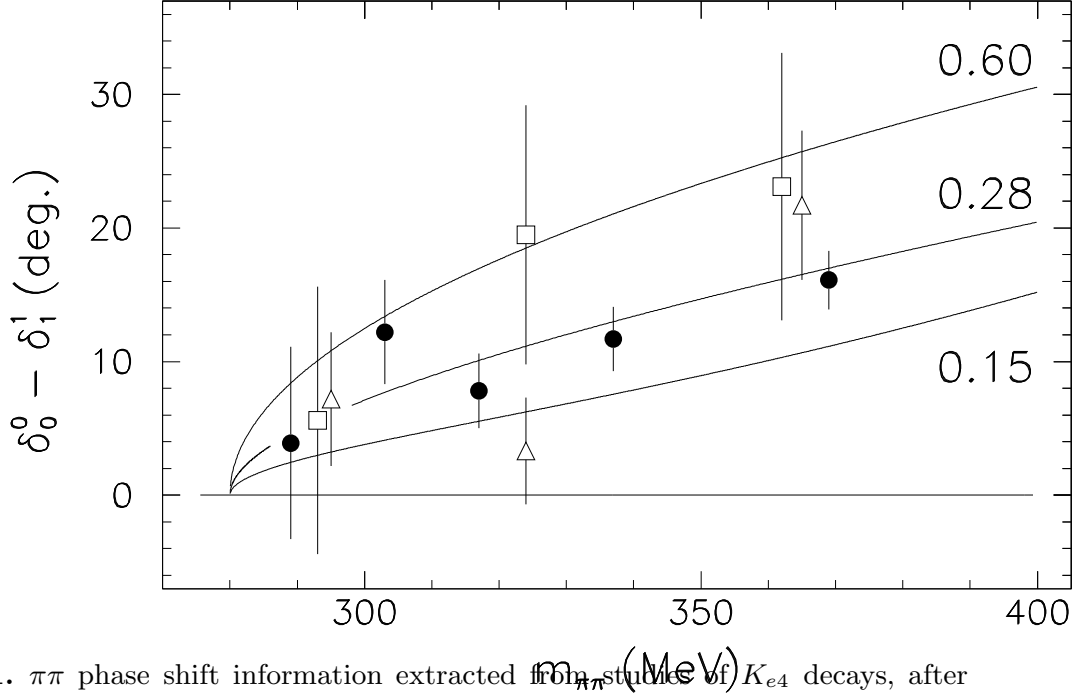


Fig. 1. $\pi\pi$ phase shift information extracted from studies of K_{e4} decays, after Rosselet *et al.* [5]. Phase-shift difference $\delta_0^0 - \delta_1^1$ is plotted against $m_{\pi\pi}$, the dipion invariant mass. Full circles represent results of Rosselet *et al.* [5], while open squares and triangles represent the results of Zylberstejn [41] and Beier *et al.* [42], respectively. The three curves correspond to phase shift solutions assuming three different values of a_0^0 , as noted.

By itself, the Geneva–Saclay experiment determines a_0^0 with a $\sim 35\%$ uncertainty, and constrains b :

$$a_0^0 = 0.31 \pm 0.11 \mu^{-1} \quad , \quad b = b_0^0 - a_1^1 = 0.11 \pm 0.16 \mu^{-1} \quad ,$$

where b_0^0 is the s-wave $I = 0$ slope parameter defined in the usual way by

$$\frac{\text{Re}A_l^I}{q^{2l}} = a_l^I + b_l^I q^2 + O(q^4) \quad , \quad \text{with} \quad q = \frac{1}{2} \sqrt{s - 4\mu^2} \quad , \quad (1)$$

where A_l^I is the $\pi\pi$ partial wave amplitude, s is the center of mass energy of the two pions, and $\mu \equiv m_\pi$ is the pion mass.

$\pi\pi$ scattering phase shifts are, however, further constrained by unitarity, analyticity, crossing and Bose symmetry, extensively studied by Roy [6], and Basdevant *et al.* [7,8]. These constraints are expressed in a set of dispersion relations known as the “Roy equations”, which have been evaluated on the basis of existing peripheral $\pi N \rightarrow \pi\pi N$ data (see Sect. 2.2). By applying the Roy equation constraints of Basdevant, *et al.* [8], and thus combining the K_{e4} and peripheral pion production results, more accurate values for a_0^0 and b_0^0 were obtained [5], as well as values of scattering lengths and slope parameters for $(l = 0, I = 2)$ and $(l = 1, I = 1)$ [9].

The present experimental accuracy of the K_{e4} measurement of the $\pi\pi$ phase shifts clearly needs to be improved. It is also evident that an independent accurate experimental determination of a_0^2 , the $I = 2$ s-wave scattering length, is called for, since this quantity is not directly constrained by K_{e4} measurements.

2.2 Peripheral $\pi N \rightarrow \pi\pi N$ Reactions: the Chew–Low Method

Particle production in peripheral collisions can be used to extract information on the scattering of two of the particles in the final state, as shown by Chew and Low in 1959 [10]. Applied to the $\pi N \rightarrow \pi\pi N$ reaction, the well-known Chew–Low formula,

$$\sigma_{\pi\pi}(m_{\pi\pi}) = \lim_{t \rightarrow \mu^2} \left[\frac{\partial^2 \sigma_{\pi\pi N}}{\partial t \partial m_{\pi\pi}} \cdot \frac{\pi}{\alpha f_\pi^2} \cdot \frac{p^2(t - \mu^2)^2}{t m_{\pi\pi} k} \right] , \quad (2)$$

relates $\sigma_{\pi\pi}(m_{\pi\pi})$, the cross section for pion-pion scattering, to double differential $\pi N \rightarrow \pi\pi N$ cross section and kinematical factors: p , momentum of the incident pion, $m_{\pi\pi}$, the dipion invariant mass, t , the Mandelstam square of the 4-momentum transfer to the nucleon, $k = (m_{\pi\pi}^2/4 - \mu^2)^{1/2}$, momentum of the secondary pion in the rest frame of the dipion, $f_\pi \approx 93$ MeV, the pion decay constant, and $\alpha = 1$ or 2 , a statistical factor involving the pion and nucleon charge states. The method relies on an accurate extrapolation of the double differential cross section to the pion pole in order to isolate the one pion exchange (OPE) pole term contribution. Since the exchanged pion is off-shell in the physical region ($t < 0$), this method requires measurements under conditions which maximize the OPE contribution and minimize all background contributions. Thus, suitable measurements require peripheral pion production at values of t as close to zero as possible, which become available at incident momenta typically above ~ 3 GeV/c.

The Chew–Low method has been refined considerably over time, particularly by Baton and coworkers [11], whose approach enables extraction of $\pi\pi$ phase shifts through appropriate treatment of the angular dependence of the $\pi N \rightarrow \pi\pi N$ exclusive cross sections. Crossing, Bose and isospin symmetries, analyticity and unitarity, provide dispersion relation constraints on

the $\pi\pi$ phase shifts, the “Roy equations” [6-8]. These constraints are particularly useful in evaluating $\pi\pi$ scattering lengths because available phase space restricts severely the statistics of peripheral $\pi N \rightarrow \pi\pi N$ measurements below $m_{\pi\pi} \approx 500$ MeV, while accurate data are available at higher $\pi\pi$ energies.

The data base for these analyses has essentially not changed since the early 1970’s, and is dominated by two experiments, performed by the Berkeley [12] and CERN-Munich [13] groups. The latter of the two measurements has much higher statistics (300 k events compared to 32 k in the Berkeley experiment). These data were subsequently analyzed by numerous authors, too many to review here; ultimately, the peripheral pion production results were combined with the Geneva-Saclay K_{e4} data in a comprehensive dispersion-relation analysis [9], as discussed in Sect. 2.1.

There have been independent Chew–Low type analyses since the 1970’s; the last published one, performed by the Kurchatov Institute group in 1982, was based on a set of some 35,000 events recorded in bubble chambers [14]. The same group has recently updated their analysis [15].

2.3 $\pi N \rightarrow \pi\pi N$ Reactions near Threshold

Early on, Weinberg pointed out that the OPE graph dominates the $\pi N \rightarrow \pi\pi N$ reaction at threshold [4]. Subsequently, Olsson and Turner constructed a soft-pion lagrangian containing only the OPE and contact terms at threshold [16], and derived a straightforward relation between the $\pi\pi$ and $\pi N \rightarrow \pi\pi N$ threshold amplitudes with only one parameter, ξ , the chiral symmetry breaking parameter. Thus, in Olsson–Turner’s model, it is sufficient to measure total $\pi N \rightarrow \pi\pi N$ cross sections, from which quasi-amplitudes can be calculated and extrapolated to threshold. In spite of recent strong criticism for incompatibility with QCD and oversimplified dynamical assumptions, Olsson and Turner’s work to date provides the sole direct relation between $\pi N \rightarrow \pi\pi N$ observables and the $\pi\pi$ lagrangian. In this way, soft-pion theory has provided the main inspiration for the near-threshold $\pi N \rightarrow \pi\pi N$ measurements and, in spite of its shortcomings, is still being used by experimentalists to relate the results from different reaction channels in a systematic way.

Unlike the methods described in Sects. 2.1 and 2.2, the last ten years have witnessed a great deal of experimental activity on exclusive and inclusive near-threshold $\pi N \rightarrow \pi\pi N$ measurements. As in Chew–Low peripheral pion production, there are 5 charge channels accessible to measurement,

$$\begin{aligned} \pi^- p \rightarrow \pi^- \pi^+ n \quad , \quad \pi^- p \rightarrow \pi^0 \pi^0 n \quad , \quad \pi^- p \rightarrow \pi^- \pi^0 p \quad , \\ \pi^+ p \rightarrow \pi^+ \pi^0 p \quad , \quad \text{and} \quad \pi^+ p \rightarrow \pi^+ \pi^+ n \quad . \end{aligned}$$

For brevity, we label them with their final state charges as $(-+n)$, ..., $(++n)$, respectively. Recent experiments reporting total $\pi N \rightarrow \pi\pi N$ cross

sections are summarized below, while data available before 1984 is reviewed in Ref. [17].

The OMICRON group at CERN has measured cross sections in the $(+-n)$, $(-0p)$, $(++n)$ and $(+0p)$ channels [18]. They detected coincident pairs of charged particles in a two-sided magnetic spectrometer, restricted to in-plane kinematics. (Limitations inherent in the extraction of total cross sections from data restricted to in-plane kinematics were recently discussed by the Erlangen group [19].) The thin gas target, limited magnetic spectrometer acceptance, and background subtraction, result in large error bars for some of the low-energy OMICRON data points, particularly in the $(++n)$ channel.

At TRIUMF, Sevier and coworkers measured inclusive cross sections for the reaction $(++n)$ using a novel technique involving an active plastic scintillator target combined with neutron detection [20]. Total cross sections were evaluated assuming s-wave dominance due to the proximity of the threshold (5 MeV below their lowest energy measurement). It is significant that the TRIUMF results disagree with the $(++n)$ OMICRON data.

J. Lowe and coworkers measured the $(00n)$ channel at Brookhaven using the Crystal Box detector [21]. Due to the large solid-angle coverage of the detector, this was a kinematically complete measurement of 4 photons following the decays of the two π^0 's in the final state. As all particles in the final state are neutral, the lowest point was also about 5 MeV above the threshold.

Finally, a Virginia-Stanford-LAMPF team studied the $(+0p)$ channel using the LAMPF π^0 spectrometer and an array of plastic scintillation telescopes for π^+ and p detection [22,23]. Three classes of exclusive events were recorded simultaneously: $\pi^+\pi^0$ and π^0p double coincidences, and $\pi^+\pi^0p$ triple coincidences. Since the acceptance of the apparatus and the backgrounds were significantly different for the three classes of events, this experiment had a strong built-in consistency check.

Published total cross sections for all five channels are summarized in Fig. 2., shown in the form of quasi-amplitudes, $|a(\pi\pi N)|$, extracted from the total cross sections following the prescription of Olsson and Turner [16]

$$\sigma(\pi N \rightarrow \pi\pi N) = |a(\pi\pi N)|^2 \cdot \alpha \cdot p_\pi^2 \times \text{phase space} \quad , \quad (3)$$

where p_π is the c.m. incident pion momentum, and the statistical factor $\alpha = 1/2$ for the $(++n)$ and $(00n)$ channels, and $\alpha = 1$ for the other three channels.

Apart from the pronounced disagreement in the $(++n)$ channel between the TRIUMF [20] and OMICRON [18] data, represented in Fig. 2. by full circles and full triangles, respectively, the entire body of data appears globally consistent within the quoted uncertainties. The straight lines drawn through the data in Fig. 2 are the result of a constrained soft-pion analysis following Olsson and Turner performed by the Virginia group, with

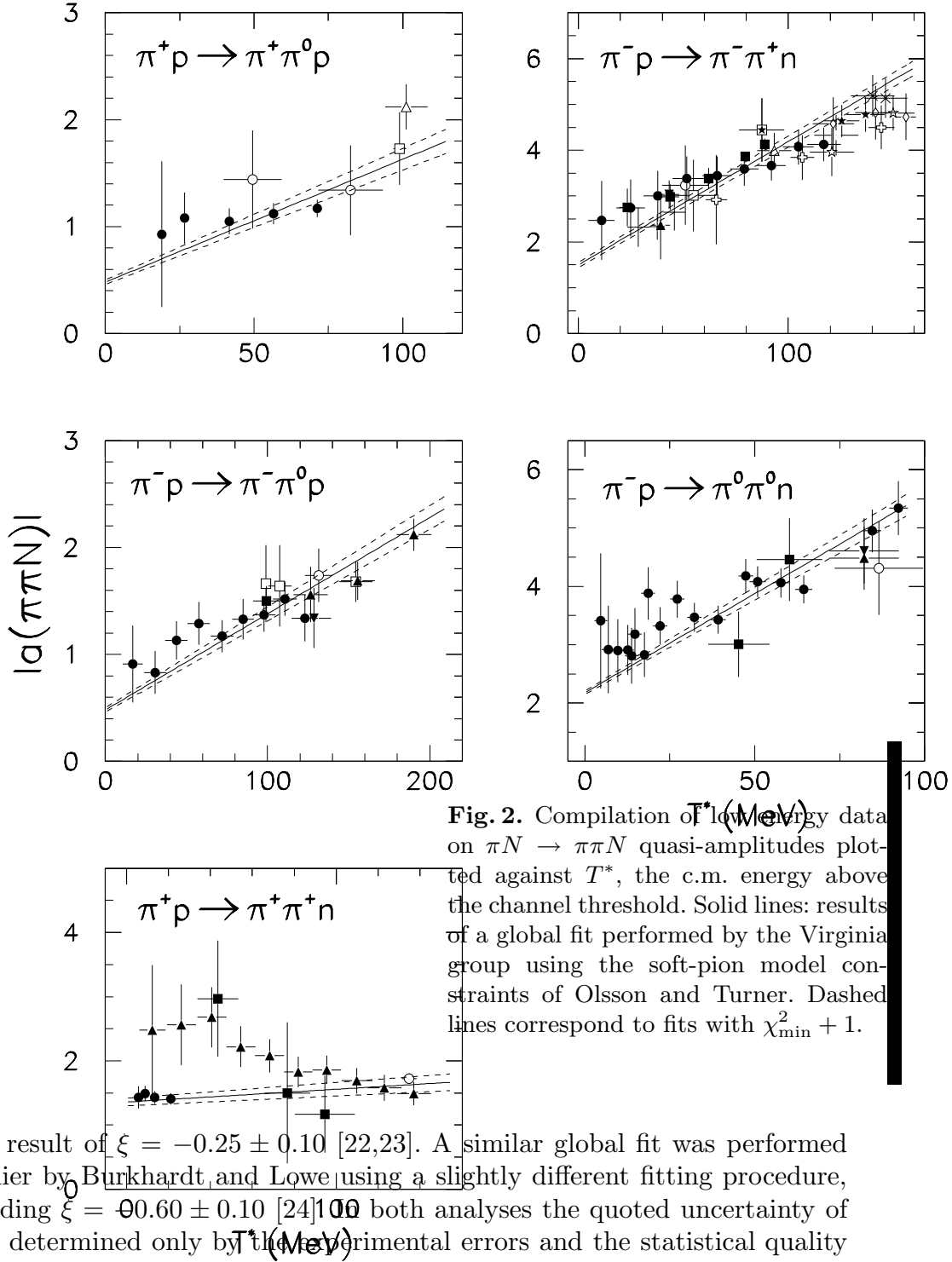
GLOBAL FIT OF $\pi\pi N$ THRESHOLD AMPLITUDES

Fig. 2. Compilation of low-energy data on $\pi N \rightarrow \pi\pi N$ quasi-amplitudes plotted against T^* , the c.m. energy above the channel threshold. Solid lines: results of a global fit performed by the Virginia group using the soft-pion model constraints of Olsson and Turner. Dashed lines correspond to fits with $\chi^2_{\min} + 1$.

the result of $\xi = -0.25 \pm 0.10$ [22,23]. A similar global fit was performed earlier by Burkhardt and Lowe using a slightly different fitting procedure, yielding $\xi = -0.60 \pm 0.10$ [24]. In both analyses the quoted uncertainty of ξ is determined only by the experimental errors and the statistical quality of the global fit. We can interpret the spread between the two values as due to the systematic uncertainties of the method, and take the mean as a representative soft-pion analysis result.

The body of near-threshold $\pi N \rightarrow \pi\pi N$ data keeps growing. Several experiments presently under way at TRIUMF and LAMPF are expected to

yield new results shortly, on both exclusive and inclusive cross sections in the $(+-n)$, $(++n)$, $(+0p)$ and $(-0p)$ channels. We also note the high-statistics angular correlation measurements in the $(+-n)$ channel performed by the Erlangen group, who did not report total cross sections [25].

Closer analysis of the exclusive cross sections is only now beginning, as high-statistics data have not been available until recently, and the methods of analysis are still being refined. In a series of recent papers, the Erlangen group has focused on the main graphs contributing to the continuum $\pi N \rightarrow \pi\pi N$ amplitude [26]. On the other hand, the St. Petersburg group [27] has derived the most general constraints on the kinematical dependence of the background and the OPE amplitudes in the physical region, as dictated by basic symmetries: crossing, Bose, isospin, analyticity and unitarity.

3 Comparison of Experimental Results and Predictions

In order to obtain a proper perspective on the existing data and the three experimental methods discussed in the preceding sections, we review briefly the various theoretical calculations of the s-wave $\pi\pi$ scattering lengths, in the order in which they appeared.

Weinberg's soft-pion model of chiral symmetry breaking relied on current algebra and PCAC [4,28]. Weinberg required of his lagrangian that $\partial^\mu A_\mu$, the divergence of the axial-vector current, form a chiral quadruplet with the pion field. Consequently, the $\pi\pi$ part of the lagrangian assumes the form:

$$\mathcal{L}_{\pi\pi} = -\frac{1}{4f_\pi^2} \cdot [\phi^2(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2(\phi^2)^2] \quad , \quad (4)$$

where ϕ is the pion field. Weinberg also noted that the s-wave scattering lengths a_0^0 and a_0^2 are constrained linearly:

$$2a_0^0 - 5a_0^2 = 6 \frac{\mu}{8\pi f_\pi^2} \approx 0.56 \mu^{-1} \quad . \quad (5)$$

Predictions by Schwinger [29] and Chang and Gürsey [30] differed from Weinberg's in the form of the response of the pion field to chiral transformations, resulting in different coefficients of the ϕ^4 pion mass term in (4). Instead of the Weinberg's coefficient 1/2, Schwinger's model suggests 1/4, and Chang and Gürsey's 1/3, respectively. Consequently, calculated values for the $\pi\pi$ s-wave scattering lengths differ.

Since in soft-pion theory (5) constrains a_0^0 and a_0^2 linearly, different models need only to fix the ratio a_0^2/a_0^0 , i.e., only one degree of freedom remains. In this respect, Olsson and Turner's parameter ξ (see Sect. 2.3 and Ref. [16]) determines the magnitude of the ϕ^4 pion mass term in (4), and, consequently, the $\pi\pi$ scattering lengths. Thus, Weinberg's prediction is

equivalent to setting $\xi = 0$, Schwinger's and Chang-Gürsey's to $\xi = 1$ and $2/3$, respectively. Although QCD has confirmed Weinberg's choice as the correct leading-order term, we include the two latter results for historical completeness.

The current-algebra calculation of Weinberg was improved in 1982 by Jacob and Scadron who introduced a correction due to the non-soft $S^* \rightarrow \pi\pi$ isobar background [31]. At about the same time, Gasser and Leutwyler calculated the $\pi\pi$ scattering amplitude to order p^4 in ChPT [32], and gave scattering length predictions. Also inspired by QCD, Ivanov and Troitskaya used the model of dominance by quark loop anomalies (QLAD) to obtain $\pi\pi$ scattering lengths [33]. On the other hand, the Jülich group constructed a dynamical model of pseudoscalar-pseudoscalar meson scattering based on meson exchange, and applied it to calculate a number of $\pi\pi$ and $K\pi$ scattering observables at low and intermediate energies [34]. A somewhat complementary approach to ChPT is the Nambu-Jona-Lasinio model [3]. Calculations of $\pi\pi$ scattering lengths within the $SU(2) \times SU(2)$ and $SU(3) \times SU(3)$ realizations of the NJL model are found in Refs. [35] and [36], respectively.

Most recently, Kuramashi and coworkers successfully applied quenched lattice QCD on a $12^3 \times 20$ lattice, and obtained $I = 0$ and 2 $\pi\pi$ scattering length values in the neighborhood of the older current-algebra calculations [37]. Finally, Roberts *et al.* have recently developed a model field theory, referred to as the global color-symmetry model (GCM), in which the interaction between quarks is mediated by dressed vector boson exchange [38]. The model incorporates dynamical chiral symmetry breaking, asymptotic freedom, and quark confinement, and was applied to calculate a number of low-energy observables in the pion sector of QCD.

Table 1. summarizes quantitatively the theoretical model predictions and the experimental determinations of the s-wave $\pi\pi$ scattering lengths. The same quantities are also displayed in Fig. 3.

Considerable scatter of predicted values of a_0^0 and a_0^2 is evident in Fig. 3. Even disregarding the 1960's calculations of Schwinger [29] and Chang and Gürsey [30] which were superseded by QCD, a significant range of predicted values remains. However, in view of the present experimental uncertainties (see Fig. 3.), most authors claim that their results are supported by the data. This is not a satisfactory state of affairs. Progress is required on two fronts in order to improve the present uncertainties in the experimentally derived $\pi\pi$ scattering lengths.

First, more accurate data on K_{e4} decays are needed, in order to reduce the current error limits in the analysis. However, K_{e4} data alone will not suffice because of their insensitivity to the $I = 2$ $\pi\pi$ channel. Measurements of the $\pi^+\pi^-$ atom proposed at several laboratories, if feasible with reasonable statistics, could provide the much needed additional theoretically unambiguous information. The quantity to be measured is the decay rate

Table 1. The s-wave $\pi\pi$ scattering lengths: compilation of theoretical model predictions and of experimental analysis results. The fourth column lists the chiral symmetry breaking “offset” $2a_0^0 - 5a_0^2$ defined in Eq. (5). Uncertainties were not entered for theoretical predictions, as they are normally not quoted by authors. For the few calculations that did estimate uncertainties, they ranged from ~ 5 to $\sim 10\%$. All values are listed in units of inverse pion mass, μ^{-1} .

Model/Method	a_0^0 (μ^{-1})	a_0^2 (μ^{-1})	$2a_0^0 - 5a_0^2$	Ref.
<i>(a) Theoretical model predictions</i>				
Weinberg	0.16	-0.046	0.56	[4]
Schwinger	0.10	-0.069	0.56	[29]
Chang-Gürsey	0.12	-0.062	0.56	[30]
Jacob-Scadron	0.20	-0.029	0.56	[31]
ChPT	0.20	-0.042	0.61	[32]
QLAD	0.21	-0.060	0.72	[33]
Meson exch'ge	0.31	-0.027	0.76	[34]
NJL – SU(2)	0.22	-0.074	0.81	[35]
NJL – SU(3)	0.26	-0.062	0.83	[36]
Lattice QCD	0.22	-0.042	0.65	[37]
GCM (fit 1)	0.16	-0.042	0.53	[38]
GCM (calc.)	0.17	-0.048	0.58	[38]
<i>(b) Results of experimental analyses</i>				
K_{e4} + Roy eq.	0.26 ± 0.05	-0.028 ± 0.012	0.66 ± 0.12	[5,9]
Chew-Low PSA	0.24 ± 0.03	-0.04 ± 0.04	0.68 ± 0.21	[14]
Soft-pion/O-T	0.188 ± 0.016	-0.037 ± 0.006	0.56 ± 0.04	[24,22]

of the $\pi^+\pi^-$ ground state into the $\pi^0\pi^0$ channel, which is proportional to $|a_0^0 - a_0^2|^2$ [39].

Second, the mounting volume and accuracy of the exclusive and inclusive near-threshold $\pi N \rightarrow \pi\pi N$ data contain, in principle, valuable information regarding threshold $\pi\pi$ scattering. Theoretical interpretation of this data requires much improvement in order to make full use of this information. The work of Bernard, Kaiser and Meißner [40] appears to be a promising step in that direction.

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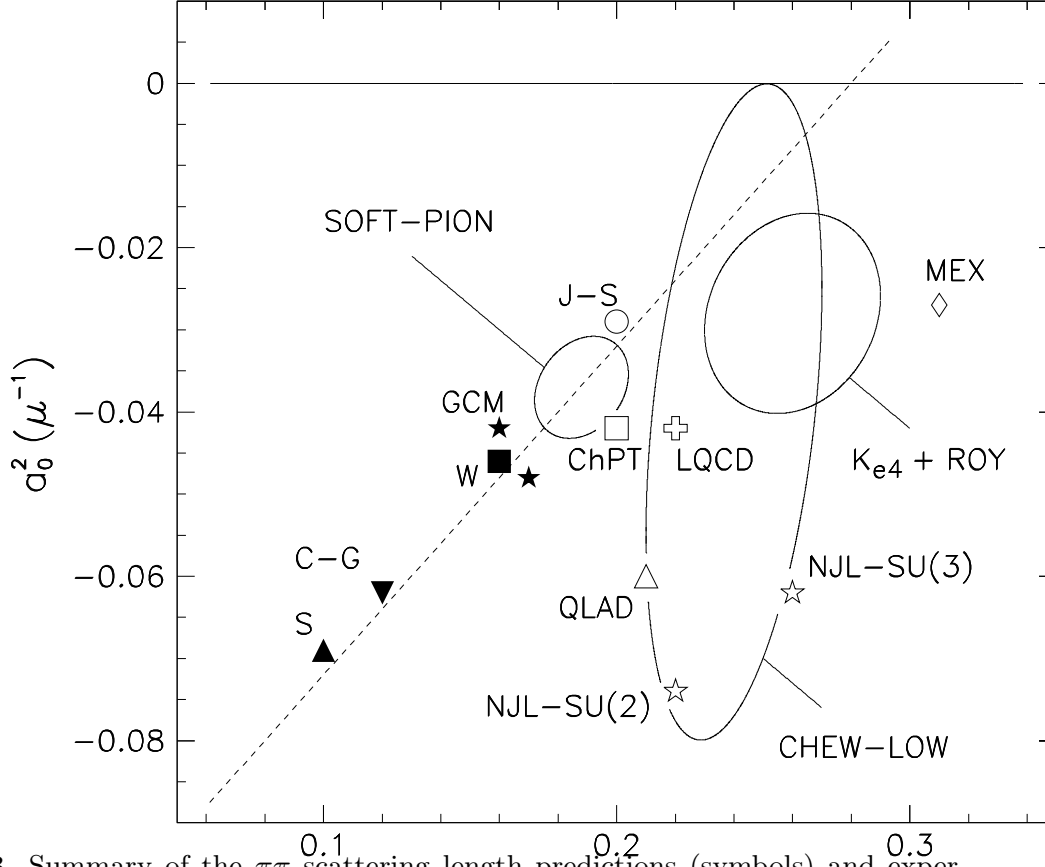


Fig. 3. Summary of the $\pi\pi$ scattering length predictions (symbols) and experimental results (contour limits). Dashed line: Weinberg's constraint given in Eq. (5). Numerical values of $\pi\pi$ scattering lengths, experimental limits, and corresponding references are listed in Table 1. Model calculations: Weinberg (full square), Schwinger (filled triangle), Chang and Gürsey (filled inverted triangle), Jacob and Scadron (open circle), Gasser and Leutwyler – ChPT (open square), Ivanov and Troitskaya – QLAD (open triangle), Lohse *et al.* – Meson Exchange (open rhomb), Ruivo *et al.* and Bernard *et al.* – NJL (open stars), Kuramashi *et al.* – quenched lattice gauge QCD (open cross), Roberts *et al.* – GCM (filled stars).

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